

Figure 1: Computer-aided proof of Theorem 3 in graphical form, showing that there is no Condorcet extension that satisfies participation for $m \geq 4$ and $n \geq 12$. See Section 4 for an explanation of how to read this diagram.

(and indeed a tight) bound, and additionally exhibits a lot of symmetry that was also present in the MUS we extracted.

6. MAIN RESULT

We are now in a position to state and prove our main claim that Condorcet extensions cannot avoid the no-show paradox for 12 or more voters (when there are at least 4 alternatives), and that this result is optimal.

THEOREM 3. *There is no Condorcet extension that satisfies participation for $m \geq 4$ and $n \geq 12$.*

PROOF. The proof follows the structure depicted in Figure 1. Let R be the preference profile shown there.

Since R remains fixed after relabelling alternatives according to $\mathbf{abcd} \mapsto \mathbf{dcba}$, we may assume without loss of generality that $f(R) \in \{\mathbf{a}, \mathbf{b}\}$. (An explicit proof in case $f(R) \in \{\mathbf{c}, \mathbf{d}\}$ is indicated in Figure 1.)

By participation, it follows from $f(R) \in \{\mathbf{a}, \mathbf{b}\}$ that also $f(R_\alpha := R + 2 \cdot \mathbf{abcd}) \in \{\mathbf{a}, \mathbf{b}\}$ since the voters with preferences \mathbf{abcd} cannot be worse off by joining the electorate. If $f(R_\alpha) = \mathbf{a}$, again by participation, removing 2 voters with preferences \mathbf{bdca} does not change the winning alternative (so $f(R_\alpha - 2 \cdot \mathbf{bdca}) = \mathbf{a}$), and neither does adding \mathbf{acdb} , so $f(R_\alpha - 2 \cdot \mathbf{bdca} + \mathbf{acdb}) = \mathbf{a}$, which, however, is in conflict with $R_\alpha - 2 \cdot \mathbf{bdca} + \mathbf{acdb}$ having a Condorcet winner, \mathbf{c} .

Thus we must have $f(R_\alpha) = \mathbf{b}$, which implies that $f(R_\alpha - \mathbf{dcab}) = \mathbf{b}$, and thus $f(R_\beta := R_\alpha - \mathbf{dcab} - 2 \cdot \mathbf{cabd}) \in \{\mathbf{b}, \mathbf{d}\}$.

We again proceed by cases: If $f(R_\beta) = \mathbf{b}$, we can add a voter \mathbf{badc} to obtain a profile with Condorcet winner \mathbf{a} , which contradicts participation. But then, if $f(R_\beta) = \mathbf{d}$, we get that $f(R_\beta - \mathbf{abcd}) = \mathbf{d}$ and, by another application of participation, that $f(R_\beta - \mathbf{abcd} + 3 \cdot \mathbf{dcba}) = \mathbf{d}$ in contrast to the existence of Condorcet winner \mathbf{b} , a contradiction.

If $m > 4$, we add bad alternatives $\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_{m-4}$ to the bottom of R and all other voters. By Lemma 1, f chooses from $\{\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}\}$ at each step, completing the proof. \square

The following result establishes that our bound on the number of voters is tight. A very useful feature of our computer-aided approach is that we can easily add additional requirements for the desired voting rule. Two common requirements for voting rules are that they should only

return alternatives that are *Pareto-optimal* and contained in the *top cycle* (also known as the *Smith set*) (see, e.g., [17]).

THEOREM 4. *There is a Condorcet extension f that satisfies participation for $m = 4$ and $n \leq 11$. Moreover, f is pairwise, Pareto-optimal, and a refinement of the top cycle.*

The Condorcet extension f is given as a look-up table, which is derived from the output of a SAT solver. The look-up table lists all 1,204,215 weighted tournaments inducible by up to 11 voters and assigns each an output alternative (see Figure 2 for an excerpt). The relevant text file has a size of 28 MB (gzipped 4.5 MB) and is available as part of an arXiv version of this paper [10].

Comparing this voting rule with known voting rules, it turns out that it picks a maximin winner in 99.8% and a Kemeny winner in 98% of all weighted tournaments. Moreover, the rule agrees with the maximin rule with lexicographic tie-breaking on 95% of weighted tournaments. The similarity with the maximin rule is interesting insofar as a well-documented flaw of the maximin rule is that it fails to be a refinement of the top cycle (and may even return Condorcet losers). Our computer-generated rule always picks from the top cycle while it remains very close to the maximin rule.

80% of the considered weighted tournaments admit a Condorcet winner, which uniquely determines the output of the rule; this can be used to reduce the size of the look-up table.

7. SET-VALUED VOTING RULES

A drawback of voting rules, as we defined them so far, is that the requirement to always return a single winner is in conflict with basic fairness conditions, namely anonymity and neutrality. A large part of the social choice literature therefore deals with set-valued voting rules, where ties are usually assumed to be eventually broken by some tie-breaking mechanism.

A *set-valued voting rule* (sometimes known as a voting *correspondence* or as an *irresolute* voting rule) is a function $F: \mathcal{R}^{\mathcal{N}} \rightarrow 2^A \setminus \{\emptyset\}$ that assigns each preference profile R a non-empty set of alternatives. The function F is a (*set-valued*) *Condorcet extension* if for every preference profile R that admits a Condorcet winner \mathbf{x} , we have $F(R) = \{\mathbf{x}\}$.

Following the work of Pérez [29] and Jimeno et al. [22], we seek to study the occurrence of the no-show paradox in

a,#1,(1,1,1,1,1,1)	a,#11,(9,11,3,9,1,-9)
a,#1,(1,1,1,1,1,-1)	a,#11,(11,9,3,7,1,-9)
a,#1,(1,1,1,-1,1,1)	c,#11,(5,-9,-1,-11,-1,7)
a,#1,(1,1,1,-1,-1,1)	c,#11,(5,-9,-1,-11,-1,5)
a,#1,(1,1,1,1,-1,-1)	c,#11,(3,-11,-1,-9,1,7)
a,#1,(1,1,1,-1,-1,-1)	c,#11,(3,-11,-3,-9,1,7)
b,#1,(-1,1,1,1,1,1)	c,#11,(3,-11,-3,-11,-1,7)
b,#1,(-1,1,1,1,1,-1)	b,#11,(-1,3,-5,-3,5,-3)
b,#1,(-1,-1,1,1,1,1)	b,#11,(-3,3,-7,-3,5,-3)
b,#1,(-1,-1,-1,1,1,1)	b,#11,(-3,1,-7,-3,5,-3)
b,#1,(-1,1,-1,1,1,-1)	c,#11,(-3,1,-5,-5,5,-1)
b,#1,(-1,-1,-1,1,1,-1)	a,#11,(3,7,11,-3,9,11)
c,#1,(1,-1,1,-1,1,1)	a,#11,(3,7,11,-3,9,9)
c,#1,(1,-1,1,-1,-1,1)	a,#11,(3,7,11,-5,9,11)

Figure 2: Excerpt of look-up table giving a pairwise Condorcet extension satisfying participation for $n \leq 11$ voters (from Theorem 4). Each row lists a weighted tournament as $(g_R(a, b), g_R(a, c), g_R(a, d), g_R(b, c), g_R(b, d), g_R(c, d))$ with a chosen alternative, and with the number of voters inducing the tournament.

this setting. To do so, we need to define appropriate notions of participation, and for this we will need to specify agents’ preferences over *sets* of alternatives. Here, we use the *optimistic* and *pessimistic preference extensions*. An optimist prefers sets with better most-preferred alternative, while a pessimist prefers sets with better least-preferred alternative. For example, if $U = \{a, b, d\}$ and $V = \{b, c\}$, then an optimist with preferences $abcd$ prefers U to V , while a pessimist prefers V to U . With these notions, we extend the participation property to set-valued voting rules.

DEFINITION 3. A set-valued voting rule F satisfies optimistic participation if $\max_{\succsim_i} F(R + \succsim_i) \succsim_i \max_{\succsim_i} F(R)$. A set-valued voting rule F satisfies pessimistic participation if $\min_{\succsim_i} F(R) \succsim_i \min_{\succsim_i} F(R - i)$.

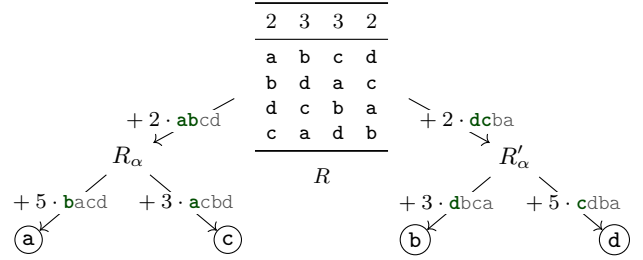
A set-valued voting rule F is called *resolute* if it always selects a single alternative, so that for all R we have $|F(R)| = 1$. A (single-valued) voting rule f is naturally identified with a resolute set-valued voting rule F ; if f satisfies participation, then this F satisfies both optimistic and pessimistic participation. Hence, by Theorem 4, there is a (resolute) set-valued Condorcet extension F that satisfies both optimistic and pessimistic participation. However, there might be hope that allowing voting rules to be irresolute also allows for participation to be attainable for more voters, and indeed this is the case.

THEOREM 5. There is a set-valued Condorcet extension F that satisfies optimistic participation for $m = 4$ and $n \leq 16$, and also is Pareto-optimal and a refinement of the top cycle.

The SAT solver indicates that no such set-valued voting rule is pairwise. Theorem 5 is optimal in the sense that optimistic participation cannot be achieved if we allow for one more voter.

THEOREM 6. There is no set-valued Condorcet extension that satisfies optimistic participation for $m \geq 4$ and $n \geq 17$.

PROOF. Let F be such a function, and consider the following 10-voter profile R :



Suppose that either $a \in F(R)$ or $b \in F(R)$. (The case of $c \in F(R)$ or $d \in F(R)$ is symmetric.) Then let $R_\alpha := R + 2 \cdot abcd$. By optimistic participation, we then have either $a \in F(R_\alpha)$ or $b \in F(R_\alpha)$. If we had $a \in F(R_\alpha)$, then also $a \in F(R_\alpha + 3 \cdot acbd)$ but this profile has Condorcet winner c , and if $b \in F(R_\alpha)$ then also $b \in F(R_\alpha + 5 \cdot bacd)$ but this profile has Condorcet winner a . This is a contradiction.

This argument extends to more than 4 alternatives by appealing to a set-valued analogue of Lemma 1. \square

Inspecting Moulin’s original proof [28] shows that it also establishes an impossibility for optimistic participation (for 25 voters). Apparently unaware of this, Jimeno et al. [22] explicitly establish such a result for 27 voters and 5 alternatives. It is worth observing that each step of the proof of Theorem 6 involves *adding* voters to the current profile, and we never remove voters. In light of Definition 3, this is the reason why the proof establishes a result for optimistic participation. If we restrict ourselves to deleting voters, we obtain a result for pessimistic participation.

THEOREM 7. There is no set-valued Condorcet extension that satisfies pessimistic participation for $m \geq 4$ and $n \geq 14$. On the other hand, for $m = 4$ and $n \leq 13$, there exists such a set-valued voting rule.

PROOF SKETCH. The proof has a similar structure to the proof of Theorem 3, displayed in Figure 1. The initial profile of this proof is $R + 2 \cdot abcd + 2 \cdot dcba$, taking R to be the profile of Figure 1. We further replace proof steps in which voters are added by similar ones where voters are deleted, and invoke pessimistic participation at each such step to obtain a contradiction. \square

This result strengthens a result of Jimeno et al. [22], who show that for $m \geq 5$ no set-valued Condorcet extension satisfying a property called “weak translation invariance” can also satisfy pessimistic participation. Our proof does not need the extra assumption, already works for $m = 4$ alternatives, and uses just 14 instead of 971 voters.²

As previously observed, adding voters in our impossibility proofs corresponds to optimistic participation, while removing voters corresponds to pessimistic participation. In the proof of Theorem 3, we use both operations, which allows for a tighter bound of just 12 voters. In the set-valued setting, we can formulate this result in a slightly stronger way.

²The large number of voters is due to several applications of the “weak translation invariance” axiom, each of which adds $5! = 120$ voters to the preference profile under consideration.

THEOREM 8. *There is no set-valued Condorcet extension that satisfies optimistic and pessimistic participation simultaneously for $m \geq 4$ and $n \geq 12$. On the other hand, for $m = 4$ and $n \leq 11$ such a set-valued rule exists (and also is Pareto-optimal and a refinement of the top cycle).*

PROOF. Use the proof of Theorem 3, invoking optimistic participation for edges labelled with the addition of a voter (+), and invoking pessimistic participation for edges labelled with removal of a voter (-). On the other hand, the (single-valued) voting rule of Theorem 4 clearly satisfies both optimistic and pessimistic participation. \square

The preference extension combining the optimistic and pessimistic preference extension is also known as the *Egli-Milner extension*.

8. PROBABILISTIC VOTING RULES

A *probabilistic voting rule* (also known as a *social decision scheme*) assigns to each preference profile R a probability distribution (or *lottery*) over A . Thus, a probabilistic voting rule might assign a fair coin flip between \mathbf{a} and \mathbf{b} as the outcome of an election.

Formally, let $\Delta(A) = \{\mathbf{p} : A \rightarrow [0, 1] : \sum_{\mathbf{x} \in A} \mathbf{p}(\mathbf{x}) = 1\}$ be the set of lotteries over A ; a lottery $\mathbf{p} \in \Delta(A)$ assigns probability $\mathbf{p}(\mathbf{x})$ to alternative \mathbf{x} . A probabilistic voting rule f is a function $f : \mathcal{R}^{\mathcal{E}(N)} \rightarrow \Delta(A)$. In this context, we say that f is a *Condorcet extension* if $f(R)$ puts probability 1 on the Condorcet winner of R whenever it exists: if R admits \mathbf{x} as the Condorcet winner, then $f(R)(\mathbf{x}) = 1$.

As in the set-valued case, we need a notion of comparing outcomes in order to extend the definition of participation. Here, we use the concept of *stochastic dominance (SD)*.

DEFINITION 4. *Let $\succsim \in \mathcal{R}$ be a preference relation over A , and let $\mathbf{p}, \mathbf{q} \in \Delta(A)$ be lotteries. Then \mathbf{p} is (weakly) SD-preferred over \mathbf{q} by \succsim if for each alternative \mathbf{x} , we have*

$$\sum_{\mathbf{y} \succsim \mathbf{x}} \mathbf{p}(\mathbf{y}) \geq \sum_{\mathbf{y} \succsim \mathbf{x}} \mathbf{q}(\mathbf{y}).$$

In this case, we write $\mathbf{p} \succsim^{\text{SD}} \mathbf{q}$.

For example, the lottery $\frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{c}$ is SD-preferred to the lottery $\frac{1}{3}\mathbf{a} + \frac{1}{3}\mathbf{b} + \frac{1}{3}\mathbf{c}$ by a voter with preferences \mathbf{abcd} . A voter with preferences \mathbf{bacd} will feel the other way around. The main appeal of stochastic dominance stems from the following equivalence: $\mathbf{p} \succsim^{\text{SD}} \mathbf{q}$ if and only if \mathbf{p} yields at least as much von-Neumann-Morgenstern utility as \mathbf{q} under any utility function that is consistent with the ordinal preferences \succsim . Using this notion of comparing lotteries, we can define participation analogously to the previous settings.

DEFINITION 5. *A probabilistic voting rule f satisfies strong SD-participation if $f(R) \succsim_i^{\text{SD}} f(R-i)$ for all $R \in \mathcal{R}^N$ and $i \in N$ with $N \in \mathcal{E}(N)$.*

Any (single-valued) voting rule f can be seen as a probabilistic voting rule that puts probability 1 on its chosen alternative. If f satisfies participation, then this derived probabilistic voting rule is easily seen to satisfy strong SD-participation. Hence Theorem 4 gives us a probabilistic Condorcet extension that satisfies strong SD-participation for $n \leq 11$ voters and $m = 4$ alternatives.

We now establish a connection between strong SD-participation and the set-valued notions of participation

that we considered in Section 7. This connection will allow us to translate the impossibility results we obtained there to the probabilistic setting. To set up this connection, let us define the *support* of a lottery $\mathbf{p} \in \Delta(A)$ to be $\text{supp}(\mathbf{p}) := \{\mathbf{x} \in A : \mathbf{p}(\mathbf{x}) > 0\}$.

PROPOSITION 1. *Let f be a probabilistic voting rule satisfying strong SD-participation. Let $F = \text{supp} \circ f$ be the support of f , i.e., $F(R) = \text{supp}(f(R))$ for all profiles R . Then F satisfies both optimistic and pessimistic participation.*

PROOF. We only verify optimistic participation; the pessimistic case is similar. Let R be a preference profile with electorate $N \in \mathcal{E}(N)$, and let $i \in N \setminus N$ be a voter with preferences \succsim_i . Let $\mathbf{x} = \max_{\succsim_i} F(R)$, and let $U = \{\mathbf{y} : \mathbf{y} \succsim_i \mathbf{x}\}$. We need to show that $\max_{\succsim_i} F(R + \succsim_i) \succsim_i \mathbf{x}$, by finding an alternative $\mathbf{y} \in U$ that is in the support of $f(R + \succsim_i)$.

But since f satisfies strong SD-participation, we have

$$\sum_{\mathbf{y} \in U} f(R + \succsim_i)(\mathbf{y}) \geq \sum_{\mathbf{y} \in U} f(R)(\mathbf{y}) > 0,$$

where the strict inequality follows from the definition of the support and of \mathbf{x} . Hence some alternative from U is in the support of $f(R + \succsim_i)$, as required. \square

Putting these results together with the impossibility result of Theorem 8, we obtain the following.

THEOREM 9. *There is no probabilistic Condorcet extension that satisfies strong SD-participation for $n \geq 12$ and $m \geq 4$. On the other hand, for $m = 4$ and $n \leq 11$, such a probabilistic voting rule exists.*

Theorem 9 resolves an open problem mentioned by Brandl et al. [5, Sec. 6].

9. CONCLUSIONS AND FUTURE WORK

We have given tight results delineating in which situations no-show paradoxes must occur. As such, our results nicely complement recent advances to satisfy Condorcet-consistency and participation by exploiting uncertainties of the voters about their preferences or about the voting rule's tie-breaking mechanism [4, 5, 6].

Due to unmanageable branching factors when there are 5 alternatives (and hence $5! = 120$ possible preference relations), we were unable to check using our approach whether no-show paradoxes occur with even less voters when the number of alternatives grows. It would be interesting to gain a deeper understanding of the computer-generated Condorcet extension that satisfies participation for up to 11 voters. So far, we only know that it (slightly) differs from all Condorcet extensions that are usually considered in the literature. As a first step, it would be desirable to obtain a representation of this rule that is more concise than a look-up table.

Another interesting topic for future research is to find optimal bounds for a variant of the no-show paradox due to Sanver and Zwicker [32], in which participation is weakened to half-way monotonicity. Their proof requires 702 voters.

Acknowledgments

Christian Geist is supported by Deutsche Forschungsgemeinschaft under grant BR 2312/9-1. Dominik Peters is supported by EPSRC. Part of this work was conducted while Dominik Peters visited TUM, supported by the COST Action IC1205 on Computational Social Choice.

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